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The contract effort is theoretical and experimental investigation of (a) the nonlinear response of ships in regular and irregular waves and (b) means of controlling complicated and large-amplitude oscillations. Some of the new analytical techniques developed in applied mathematics and nonlinear dynamics have been adapted for ships motions. These techniques include perturbation techniques, bifurcation theory, renormalization techniques, Poincare′ maps, fractal concepts, knot theory, nonlinear form theory, cell-to-cell mapping, symbolic manipulators, invariant measures, and Melnikov theory. Moreover, a number of recent discoveries in nonlinear dynamics have been carried over into ship motions. For example, we have shown that the nonlinearity brings a whole range of phenomena in the rolling motion of biased and unbiased ships in regular seas. These phenomena include coexistence of attractors (long-time responses), jumps between coexisting attractors, period-multiplying bifurcations, sensitivity of response to initial conditions, chaotic motions, and capsizing. When the pitch frequency is approximately twice the roll frequency, we have shown theoretically that the ship has undesirable seakeeping characteristics, as noted by Froude in 1963. We have also shown that the ship motion can be very complicated even if the waveslopes are extremely small. The complicated motions include saturation, amplitude- and phase-modulated motions, and chaotic motions.

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FINAL REPORT
NONLINEAR SHIP DYNAMICS
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Abstract

The contract effort is theoretical and experimental investigation of (a) the nonlinear response of ships in regular and irregular waves and (b) means of controlling complicated and large-amplitude oscillations. Some of the new analytical techniques developed in applied mathematics and nonlinear dynamics have been adapted for ships motions. These techniques include perturbation techniques, bifurcation theory, renormalization techniques, Poincare′ maps, fractal concepts, knot theory, nonlinear form theory, cell-to-cell mapping, symbolic manipulators, invariant measures, and Melnikov theory. Moreover, a number of recent discoveries in nonlinear dynamics have been carried over into ship motions. For example, we have shown that the nonlinearity brings a whole range of phenomena in the rolling motion of biased and unbiased ships in regular seas. These phenomena include coexistence of attractors (long-time responses), jumps between coexisting attractors, period-multiplying bifurcations, sensitivity of response to initial conditions, chaotic motions, and capsizing. When the pitch frequency is approximately twice the roll frequency, we have shown theoretically that the ship has undesirable seakeeping characteristics, as noted by Froude in 1963. We have also shown that the ship motion can be very complicated even if the waveslopes are extremely small. The complicated motions include saturation, amplitude- and phase-modulated motions, and chaotic motions.

The following is a brief summary of the accomplishments under this contract.

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Publications:

1. A. H. Nayfeh, "Interaction of Fundamental Parametric Resonances with Subharmonic Resonances of Order One-Half," *Journal of Sound and Vibration*, Vol. 96, No. 3, 1984, pp. 333-340.

   The interaction of fundamental parametric resonances with subharmonic resonances of order one-half in a single-degree-of-freedom system with quadratic and cubic nonlinearities is investigated. The method of multiple scales is used to derive two first-order ordinary-differential equations that describe the modulation of the amplitude and the phase of the response with the nonlinearity and both resonances. These equations are used to determine the steady-state solutions and their stability. Conditions are derived for the quenching or enhancement of a parametric resonance by the addition of a subharmonic resonance of order one-half. The degree of quenching or enhancement depends on the relative amplitudes and phases of the excitations. The analytical results are verified by numerically integrating the original governing differential equation.

2. A. H. Nayfeh, "Parametric Identification of Nonlinear Dynamic Systems," *Computers & Structures*, Vol. 20. No. 1-3. 1985, pp. 487-493.

   A parametric identification technique that exploits nonlinear resonances and comparisons of the behavior of the system to be identified with those of known systems is proposed. The mathematical model is chosen in such a way that its predicted response qualitatively resembles observed responses of the physical system to chosen excitations. Moreover, instead of taking a brute-force approach that requires the simultaneous estimation of all the parameters from a given experiment, we exploit the possible resonances and design experiments, each of which

provides accurate estimates of a limited number of these parameters. Typically, these resonances owe their existence to nonlinearities in the governing equations. Experiments are proposed for the estimation of the parameters of two-degree-of-freedom systems with quadratic and cubic nonlinearities.

3. A. H. Nayfeh, "The Response of Nonlinear Single Degree of Freedom Systems to Multifrequency Excitations," *Journal of Sound and Vibration*, Vol. 102, No. 3, 1985, pp. 403-414.

The method of multiple scales is used to analyze the response of single-degree-of-freedom systems with cubic non-linearities to excitations that involve multiple frequencies. Two first-order ordinary differential equations are derived for the evolution of the amplitude and phase with damping, non-linearity, and all possible resonances. Conditions for the existence and stability of steady-state solutions are determined. These results are used to suggest simple means of controlling or minimizing the large oscillations. These means may take the form of adding non-resonant loads that shift the natural frequency of the system or adding another resonant load having the proper frequencies, amplitudes and phases.

4. A. H. Nayfeh and L. D. Zavodney. "The Response of Two-Degree-of-Freedom Systems with Quadratic Non-Linearities to a Combination Parametric Resonance," *Journal of Sound and Vibration*, Vol. 107, No. 2, 1986, pp. 329-350.

The response of two-degree-of-freedom systems with quadratic non-linearities to a combination parametric resonance in the presence of two-to-one internal resonances is investigated. The method of multiple scales is used to construct a first order uniform expansion yielding four first order non-linear ordinary differential equations governing the modulation of the amplitudes and the phases of the two modes. Steady state responses and their stability are computed for selected values of the system parameters. The effects of detuning the internal resonance, detuning the parametric resonance, the phase and magnitude of the

second mode parametric excitation, and the initial conditions are investigated. The first order perturbation solution predicts qualitatively the trivial and non-trivial stable steady state solutions and illustrates both the quenching and saturation phenomena. In addition to the steady state solutions, other periodic solutions are predicted by the perturbation amplitude and phase modulation equations. These equations predict a transition from constant steady state non-trivial responses to limit cycle responses (Hopf bifurcation). Some limit cycles are also shown to experience period doubling bifurcations. The perturbation solutions are verified by numerically integrating the governing differential equations.

5. A. H. Nayfeh and A. A. Khdeir, "Nonlinear Rolling of Ships in Regular Beam Seas," *International Shipbuilding Progress*, Vol. 33, No. 379, 1986, pp. 40-49.

A second-order approximate solution is obtained for the nonlinear established harmonic roll of a ship in regular beam seas. The perturbation solutions are compared with solutions obtained by numerically solving the nonlinear governing equations. The peak roll angle and corresponding frequency predicted by the second-order expansion are found to be in closer agreement with the numerical simulation than those predicted by the first-order expansion. Increasing the wave slope is found to increase the peak roll angle, increase the resonance width, and bend the response curves to lower frequencies. Decreasing the damping coefficients is found to increase the peak roll angle, decrease the resonance width, and also bend the response curves. Increasing the amplitude of the wave slope beyond a threshold value results in some unstable harmonic responses and a cascade of bifurcations. The roll responses experience either period doubling or period tripling bifurcations leading to chaotic behavior. A Floquet analysis is used to predict the occurrence of these bifurcations. The perturbation expansion predicts fairly well the established harmonic oscillations as well as the start of the period multiplying bifurcations.

6. A. H. Nayfeh and A. A. Khdeir, "Nonlinear Rolling of Biased Ships in Regular Beam Waves," *International Shipbuilding Progress*, Vol. 33, No. 381, 1986, pp. 84-93.

The method of multiple scales is used to determine a second-order approximate solution for the nonlinear harmonic response of biased ships in regular beam waves. A Floquet analysis is used to predict stability of stead-state harmonic responses. The perturbation solutions are compared with solutions obtained by numerical integration of the nonlinear governing roll equation. The results show that the first-order perturbation expansion may be inadequate for predicting the peak roll angle and its corresponding frequency. On the other hand, the peak established roll angle and corresponding frequency predicted by the second-order expansion are found to be in good agreement with the numerical simulation. Moreover, the perturbation expansion predicts fairly well the start of period multiplying bifurcations that lead to chaos. Biased ships are found to be more susceptible to period multiplying bifurcations and chaos than unbiased ships.

7. A. H. Nayfeh, "Perturbation Methods in Nonlinear Dynamics," in *Nonlinear Dynamics Aspects of Particle Accelerators, Lecture Notes in Physics*, No. 247, Springer-Verlag, New York, 1986, pp. 238-314.

8. A. H. Nayfeh, "Parametric Excitation of Two Internally Resonant Oscillators," *Journal of Sound and Vibration*, Vol. 119, No. 1, 1987, pp. 95-109.

The response of two-degree-of-freedom systems with quadratic nonlinearities to a principal parametric resonance in the presence of two-to-one internal resonances is investigated. The method of multiple scales is used to construct a first-order uniform expansion yielding four first-order nonlinear ordinary differential (averaged) equations governing the modulation of the amplitudes and the phases of the two modes. These equations are used to determine steady state responses and their stability. When the higher mode is excited by a principal parametric resonance, the

non-trivial steady state value of its amplitude is a constant that is independent of the excitation amplitude, whereas the amplitude of the lower mode, which is indirectly excited through the internal resonance, increases with the amplitude of the excitation. However, in addition to Poincaré-type bifurcations, this response exhibits a Hopf bifurcation leading to amplitude- and phase-modulated motions. When the lower mode is excited by a principal parametric resonance, the averaged equations exhibit both Poincaré and Hopf bifurcations. In some intervals of the parameters, the periodic solutions of the averaged equations, in the latter case, experience period-doubling bifurcations, leading to chaos.

9. A. H. Nayfeh, "On the Undesirable Roll Characteristics of Ships in Regular Seas," *Journal of Ship Research*, Vol. 32, No. 2, 1988, pp. 92-100.

In 1863 Froude observed that a ship whose frequency in pitch (heave) is twice its frequency in roll has undesirable roll characteristics. To explain this phenomenon, Paulling and Rosenberg as well as Kinney assumed the pitch (heave) motion to be a simple harmonic independent of the roll motion. Substituting the pitch (heave) expression into the roll equation, they obtained a Mathieu equation. They found that exponentially growing instabilities can occur for certain pitch amplitudes and frequency ratios. The exponential growth is unrealistic, the result of their neglecting the influence of the roll motion on the pitch (heave) motion. To improve their results, the present author offers an analysis for the nonlinear coupling of the pitch and roll modes of ship motions in regular seas. When the encounter frequency is near the pitch frequency, only the pitch mode is excited if the encountered wave amplitude (excitation amplitude) is small. As the excitation amplitude increases, the amplitude of the pitch mode increases in accordance with linear theory until it reaches a critical small value. As the excitation amplitude increases further, the pitch amplitude does not change from the critical value (that is, the pitch mode is saturated), and all the extra energy is transferred to the roll mode. Consequently, for large excitation amplitudes, the response is a combined roll and pitch motion, with the amplitude of the roll mode being very much

larger than that of the pitch mode. More dangerously, the nonlinear theory predicts instabilities in regions where the linear theory predicts stability. Moreover, the nonlinear theory predicts conditions for the nonexistence of steady-state periodic responses. Instead, the responses can be amplitude- and phase-modulated roll and pitch motions or even chaotic. When the encounter frequency is near the roll frequency, there is no saturation phenomenon and, at close to perfect resonance, there are no stead-state periodic responses in some cases. The present results indicate that large roll amplitudes are likely in this case also. Further, the results predict the possibility of large amplitudes in the roll motion even when the ship is moving through a head or following sea.

10. L. D. Zavodney and A. H. Nayfeh, "The Response of a Single-Degree-of-Freedom System with Quadratic and Cubic Nonlinearities to a Fundamental Parametric Resonance." *Journal of Sound and Vibration*, Vol. 120, No. 1, 1988, pp. 63-93.

The response of a one-degree-of-freedom system with quadratic and cubic non-linearities to a fundamental harmonic parametric excitation is investigated. The method of multiple scales is used to determine the equations that describe to second order modulation of the amplitude and phase with time about one of the foci. These equations are used to determine the fixed points and their stability. The perturbation results are verified by integrating the governing equation using a digital computer and an analogue computer. For small excitation amplitudes, the analytical results are in excellent agreement with the numerical solutions. As the amplitude of the excitation increases, the accuracy of the perturbation solution deteriorates, as expected. The large responses are investigated by using both a digital and an analogue computer. The cases of single- and double-well potentials are investigated. Systems with double-well potentials exhibit complicated dynamic behaviors including period multiplying and demultiplying bifurcations and chaos. Long-time histories, phase planes, Poincare maps, and spectra of the responses are presented.

11. A. H. Nayfeh and K. R. Asfar, "Nonstationary Parametric Oscillations," *Journal of Sound and Vibration*, Vol. 124, No. 3, 1988, pp. 529-537.

The response of a single-degree-of-freedom system with cubic non-linearity to a non-stationary principal parametric excitation is investigated. The method of multiple scales is used to derive two first-order ordinary-differential equations for the evolution of the amplitude and phase of the response. The evolution equations are numerically integrated for various sweeping rates of the amplitude and frequency of the excitation. The results show that the non-stationary response penetrates the instability regions and the higher the sweeping rate is the deeper the penetration is.

12. A. H. Nayfeh and L. D. Zavodney, "Experimental Observation of Amplitude- and Phase-Modulated Responses of Two Internally Coupled Oscillators to a Harmonic Excitation," *Journal of Applied Mechanics*, Vol. 55, 1988, pp. 706-710.

An experiment is performed on a two degree-of-freedom mechanical system having quadratic nonlinearities and linear natural frequencies ω_1 and ω_2 approximately in the ratio of two-to-one (i.e., $\omega_2 \approx 2\omega_1$). When the lower mode is excited by a harmonic excitation whose frequency Ω is nearly equal to ω_1, amplitude- and phase-modulated responses of the system have been observed for a range of the excitation frequency Ω, in qualitative agreement with the results of a second-order perturbation theory.

13. A. H. Nayfeh, "Numerical-Perturbation Methods in Mechanics," *Computers & Structures*, Vol. 30, No. 1/2, 1988, pp. 185-204.

In many nonlinear problems in mechanics, the responses are so complicated (jumps, period-multiplying bifurcations, chaos, saturation) that is is impractical if not impossible to determine their salient features by using a purely numerical technique. For weakly nonlinear systems, perturbation techniques can be used quite effectively. However, purely

analytical techniques are limited to systems with simple boundaries and composition. These limitations can be removed by combining analytical and numerical techniques. These points are illustrated by examples drawn from structural vibration, sloshing of liquids in containers, and nonlinear stability of boundary layers. The combination of analytical and numerical techniques also can be very useful for treating linear wave propagation in nonhomogeneous media. The procedure is illustrated by an example intensification and refraction of acoustic signals in partially choked converging-diverging ducts.

14. A. H. Nayfeh and N. E. Sanchez, "Bifurcations in a Forced Softening Duffing Oscillator," *International Journal of Non-Linear Mechanics*, Vol. 24, No. 6, 1989, pp. 483-497.

The response of a damped Duffing oscillator of the softening type to a harmonic excitation is analyzed in a two-parameter space consisting of the frequency and amplitude of the excitation. An approximate procedure is developed for the generation of the bifurcation diagram in the parameter space of interest. It is a combination of second-order perturbation solutions of the system in the neighborhood of its non-linear resonances and Floquet analysis. The results show that the proposed scheme is capable of predicting symmetry-breaking and period-doubling bifurcations as well as jumps to either bounded or unbounded motions. The results obtained are validated using analog- and digital-computer simulations, which show chaos and unbounded motions, among other behaviors.

15. A. H. Nayfeh and B. Balachandran, "Modal Interactions in Dynamical and Structural Systems," *Applied Mechanics Reviews*, Vol. 42, No. 11, Part 2, 1989, pp. 175-201.

We review theoretical and experimental studies of the influence of modal interactions on the nonlinear response of harmonically excited structural and dynamical systems. In particular, we discuss the response of pendulums, ships, rings, shells, arches, beam structures, surface waves, and the similarities in the qualitative behavior of these systems. The

systems are characterized by quadratic nonlinearities which may lead to two-to-one and combination autoparametric resonances. These resonances give rise to a coupling between the modes involved in the resonance leading to nonlinear periodic-quasi-periodic, and chaotic motions.

16. N. E. Sanchez and A. H. Nayfeh, "Prediction of Bifurcations in a Parametrically Excited Duffing Oscillator," *International Journal of Non-Linear Mechanics*, Vol. 25, No. 2/3, 1990, pp. 163-176.

The instability regions of the response of a damped, softening-type Duffing oscillator to a parametric excitation are determined via an algorithm that uses Floquet theory to evaluate the stability of second-order approximate analytical solutions in the neighborhood of the non-linear resonances of the system. It is shown that identification of the locus of instabilities of the periodic approximate solutions in the amplitude-frequency parameter space provides valuable information on the overall dynamic behavior of the system. The predictions are verified by using analog- and digital-computer simulations, which exhibit chaos and unbounded motions among other behaviors.

17. H. L. Neal and A. H. Nayfeh, "Response of a Single-Degree-of-Freedom System to a Nonstationary Principal Parametric Excitation," *International Journal of Non-Linear Mechanics*, Vol. 25, No. 2/3, 1990, pp. 275-284.

We examine the non-stationary response of a one-degree-of-freedom non-linear system to a non-periodic parametric excitation with varying frequency. We use the method of multiple scales to obtain equations governing the stationary and non-stationary responses of the system and we analyze the stability of the stationary responses. The response displays several phenomena, including penetration of the trivial response into the unjustable trivial region, oscillation of the response about the non-trivial stationary solution, convergence of the non-stationary response to the stationary solution, lingering of the non-trivial response into the stable trivial region, and rebounding of the non-trivial response. These

phenomena are affected by the sweep rate, the initial conditions, and the system parameters. Digital and analog computers are used to solve the original governing differential equation. The results of the simulations agree with each other and with those obtained by using the method of multiple scales.

18. A. H. Nayfeh and B. Balachandran, "Experimental Investigation of Resonantly Forced Oscillations of a Two-Degree-of-Freedom Structure," *International Journal of Non-Linear Mechanics*, Vol. 25, No. 2/3, 1990, pp. 199-209.

An experimental study of the response of a two-degree-of-freedom structure with quadratic non-linearities and a two-to-one internal resonance to an external harmonic excitation is presented. When the excitation frequency was close to the lower natural frequency of the structure, periodic, quasi-periodic, and chaotic responses were observed. Fourier spectra, time-dependent modal decompositions, and Poincare maps were used to analyze the amplitude- and phase-modulated motions of the structure.

19. A. H. Nayfeh and N. E. Sanchez, "Stability and Complicated Rolling Responses of Ships in Regular Beam Seas," *International Shipbuilding Progress*, Vol. 37, No. 412, 1990, pp. 331-352.

An analytical procedure is presented for the prediction of the stability and complicated responses of a vessel rolling in regular seas. The effectiveness of the technique is demonstrated by considering unbiased and biased ships rolling in regular beam seas, where the relative waveslope and the frequency of encounter can be alternatively changed. The procedure generates bifurcation diagrams showing the regions of the parameter space where instabilities and deterioration of seaworthiness occur so that the designer can assess the dangers and overall seaworthiness of the craft under a variety of sea conditions.

Chapters of Books

1. A. H. Nayfeh, "Parametric Excitation of Two Internally Resonant Oscillators," Proceedings of Invited Lectures and Short Communications of the 11th International Conference on Nonlinear Oscillations, Budapest, Hungary, August 17-23, 1987, pp. 181-188.

2. A. H. Nayfeh and A. A. Khdeir, "Rolling of Ships in Large-Amplitude Waves," Dynamical Systems Approaches to Nonlinear Problems in Systems and Circuits, Proceedings of the Conference on Qualitative Methods for the Analysis of Nonlinear Dynamics, New England College, Henniker, New Hampshire, June 8-13, 1986; also edited by Fathi M. A. Salam and Mark L. Levin, SIAM, 1988, pp. 290-303.

3. A. H. Nayfeh, "Application of the Method of Multiple Scales to Nonlinearly Coupled Oscillators," Chapter III, from the **Lasers, Molecules, and Methods,** Advances in Chemical Physics Volume LXXIII, edited by J. O. Hirschfelder, R. E. Wyatt, and R. D. Coalson, John Wiley & Sons, Inc., New York, 1989, pp. 137-196.

4. A. H. Nayfeh and N. E. Sanchez, "Chaos and Dynamic Instability in the Rolling Motion of Ships," Proceedings of the Seventeenth Symposium on Naval Hydrodynamics, The Hague, The Netherlands, August 29-September 2, 1988, National Academy Press, 1989, pp. 617-631.

Presentations

1. A. H. Nayfeh, "Parametric Identification of Nonlinear Dynamic Systems," Symposium on Advances and Trends in Structures and Dynamics, Washington, DC, October 22-25, 1984.

2. A. H. Nayfeh, "Perturbation Methods in Nonlinear Dynamics," a series of three invited lectures, Joint US/CERN School on Particle Accelerators, Santa Margherita di Pula, Sardinia, ITALY, January 31-February 5, 1985.

3. A. H. Nayfeh, "Quenching of One Resonance by Another," 19th Midwestern Mechanics Conference, The Ohio State University, Columbus, OH, September 9-11, 1985.

4. A. H. Nayfeh and D. T. Mook, "Saturation Phenomenon in the Response of Physical Systems to External and Parametric Excitations," 19th Midwestern Mechanics Conference. The Ohio State University, Columbus, OH. September 9-11, 1985

5. A. H. Nayfeh, "Forced Response of Nonlinearly Coupled Oscillators," UCLA, Los Angeles, CA, October 18, 1985.

6. A. H. Nayfeh and D. T. Mook, "Can the Practicing Engineer Afford to be Ignorant of Nonlinear Resonances," 56th Shock and Vibration Symposium, Monterey, CA, October 22-24, 1985.

7. A. H. Nayfeh, "Nonlinear Rolling of Ships in Beam Seas," Qualitative Methods for the Analysis of Nonlinear Dynamics Conference, Henniker, NH, June 8-13, 1986.

8. A. H. Nayfeh, "Quenching of One Resonance by Another," Annual Meeting of the Vibration Institute, Las Vegas, NV, June 24-26, 1986.

9. A. H. Nayfeh, "Forced Response of Nonlinearly Coupled Oscillators," Applied Mechanics Research Seminar No. 8, United Technologies Research Center, East Hartford, CN, September 18, 1986.

10. A. H. Nayfeh and L. D. Zavodney, "The Response of Two-Degree-of-Freedom Systems with Quadratic Nonlinearities to a Combination Parametric Resonance," with L. D. Zavodney, 57th Shock and Vibration Symposium, New Orleans, LA, October 14-16, 1986.

11 A. H. Nayfeh, "On the Undesirable Roll Characteristics of Ships in Regular Seas," DTNSRDC Seminar, Bethesda, MD, October 30, 1986.

12. A. H. Nayfeh, "Parametric Excitation of Two Internally Resonant Oscillators," Mathematics Seminar, VPI&SU, February 18, 1987.

13. A. H. Nayfeh, "Perturbation Methods in Nonlinear Dynamics," AFOSR/ARO Conference on Non-Linear Vibrations, Stability, and Dynamics of Structures and Mechanics, VPI&SU, Blacksburg, VA, March 23-25, 1987.

14. A. H. Nayfeh "Can the Practicing Engineer Afford to Be Ignorant of Nonlinear Phenomena?," Electrical Engineering Seminar, VPI&SU, May 22, 1987.

15. A. H. Nayfeh, "Can the Practicing Engineer Afford to be Ignorant of Nonlinear Phenomena?," Second Technical Workshop on Dynamics and Aeroelastic Stability Modeling of Rotorcraft Systems, Boca Raton, FL, November 18-20, 1987.

16. A. H. Nayfeh, "Analytical and Experimental Studies of Nonlinear Phenomena," 6th Annual Forum on Space Structures, Atlanta, GA, April 7-8, 1988.

17. A. H. Nayfeh, "Modal Interactions in the Nonlinear Response of Structural Elements- Theory and Experiment," Rensselaer Polytechnic Institute, Troy, NY, April 27, 1988.

18. A. H. Nayfeh and K. R. Asfar, "Nonstationary Parametric Oscillations," ASCE Engineering Mechanics Division Specialty Conference, VPI&SU, Blacksburg, VA, May 23-25, 1988.

19. A. H. Nayfeh and L. D. Zavodney, "Parametric Resonances in Nonlinear Structural Elements: Theory and Experiment," ASCE Engineering Mechanics Division Specialty Conference, VPI&SU, Blacksburg, VA, May 23-25, 1988.

20. A. H. Nayfeh and N. E. Sanchez, "Bifurcations in a Forced Softening Duffing Oscillator," Second Non-Linear Vibrations, Stability, and Dynamics of

Structures and Mechanisms Conference, VPI&SU, Blacksburg, VA, June 1-3, 1988.

21. A. H. Nayfeh and H. L. Neal, "Response of a Single-Degree-of-Freedom System to a Nonstationary Parametric Excitation - Theory and Experiment," Second Non-Linear Vibrations, Stability, and Dynamics of Structures and Mechanisms Conference, VPI&SU, Blacksburg, VA, June 1-3, 1988.

22. A. H. Nayfeh, B. Balachandran, M. A. Colbert, and M. A. Nayfeh. "Theoretical and Experimental Investigation of Complicated Responses of a Two-Degree-of-Freedom Structure," Second Non-Linear Vibrations, Stability, and Dynamics of Structures and Mechanisms Conference, VPI&SU, Blacksburg, VA, June 1-3, 1988.

23. A. H. Nayfeh and L. D. Zavodney, "Modal Interactions in the Nonlinear Response of Structural Elements- Theory and Experiment," Third International Conference on Recent Advances in Structural Dynamics, Southampton, England, July 18-22, 1988.

24. A. H. Nayfeh and N. E. Sanchez, "Nonlinear Rolling of Ships in Beam Seas," 17th Symposium on Naval Hydrodynamics, Hague, Netherlands, August 29, 1988.

25. A. H. Nayfeh, "Modal Interactions in the Nonlinear Response of Structural Elements- Theory and Experiment," University of Michigan, Ann Arbor, MI, November 16, 1988.

26. A. H. Nayfeh, "Modal Interactions in Systems with Quadratic Nonlinearities", Pan-American Congress of Applied Mechanics, Sponsored by AAM, Rio De Janeiro, Brazil, January 3-6, 1989.

Students

1. Feeny, B. F., M.S., 1986 "Identification of Nonlinear Ship Motion Using Perturbation Techniques"

2. Zavodney, L. D., Ph.D., 1987 "A Theoretical and Experimental Investigation of Parametrically Excited Nonlinear Mechanical Systems"

3. Neal, H. L., Senior Project, 1988 "Nonstationary Response of a Nonlinear System to Nonperiodic Parametric Excitations with Varying Frequency"

4. Sanchez, N. E., Ph.D., 1989 "Stability of Nonlinear Oscillatory Systems with Application to Ship Dynamics"

5. Serhan, Samir J., Ph.D., 1989, "Response of Nonlinear Structures to Deterministic and Random Excitations"

6. Balachandran, B., Ph.D., 1990 "A Theoretical and Experimental Study of Modal Interactions in Structures"